

Probabilistic formulation of a performance-based matrix combining maximum and residual deformations

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ABSTRACT: Reports from past earthquake reconnaissance observations as well as results from analytical studies indicate that most structures designed according to current codes will sustain residual (permanent) deformations in the event of a design-level earthquake, even if they perform exactly as expected. Despite this reality, little consideration is currently given to residual displacements unlike the maximum transient response during seismic design or performance assessment of structures. The concept of a performance-based matrix, where maximum and residual displacements are combined as complementary damage indicators has been recently introduced by the authors, is a viable tool to identify performance levels. In this paper, a probabilistic formulation of the performance based matrix concept is presented, within a refined framework for performance based seismic design and assessment of structures. Extensive non-linear time history analyses have been performed on equivalent SDOF systems designed using the Displacement Based Design (DBD) approach and modelled with different hysteresis rules to represent multi-storey reinforced concrete or steel buildings. Fragility curves representing the probabilities of exceedence of different joined maximum-residual performance levels are derived using bivariate probability distributions. Valuable observations are derived with regard to the contributions from the response parameters to the total probability of exceedence based on the hysteretic behaviour. As part of the proposed formulation for performance-based seismic design and assessment, performance objectives can be defined and associated with targeted probability of achieving them.

1 INTRODUCTION

Current advances in earthquake engineering favour performance based approaches for the seismic design of new structures and for the assessment and rehabilitation of existing structures located in active seismic zones [ASCE, 2000]. Recent developments in performance-based design and assessment concepts [Pampanin et al., 2002 Christopoulos and Pampanin, 2004], have highlighted the limitations and inconsistencies related to current Performance Based Seismic Engineering (PBSE) approaches, whereby the performance of a structure is typically assessed using one or multiple structural response indices, as the maximum deformation (i.e. interstorey drift or ductility) and/or cumulative inelastic energy absorbed during the earthquake. Reports from past earthquake reconnaissance observations, from shake table tests, as well as from analytical studies, indicate that most structures designed according to current codes will sustain residual deformations in the event of design level earthquake, even if they perform exactly as expected. In the framework for performance-based design and assessment presented by [Pampanin et al., 2002], particular focus has been given to the critical role of residual (permanent) deformations as a major additional and complementary damage indicator.

Earlier studies by Priestley [1993] as well as MacRae and Kawashima [1997] have discussed the importance of residual deformations when assessing the performance of structures by emphasizing the difficulty and cost associated with straightening structures after a major earthquake before repairs could be carried out. A number of researchers [MacRae and Kawashima, 1997; Borzi et al., 2001; Christopoulos et al., 2003] have then identified the post yielding stiffness as the main parameter

influencing the residual deformations of non-linear Single Degree of Freedom (SDOF) oscillators. A first attempt to introduce the residual deformation/drift as a complementary parameter in a design guidelines or code provisions is found in the 1996 Japanese seismic design specifications for highway bridges, which, as reported by Kawashima [1997], imposes an additional design check for important bridges in terms of residual displacements which are required to be smaller than 1% of the bridge height. In recent draft guidelines for performance evaluation of earthquake resistant reinforced concrete buildings under preparation by the Architectural Institute of Japan (AIJ), limits on residual crack widths are tentatively indicated and associated to ranges of maximum drift/ductility and damage level.

A residual deformation index (RDDI), which measures the degree of permanent deformations and drifts of SDOF or MODF structures, has been proposed in [Pampanin et al., 2002, 2003 and Christopoulos et al. 2003] as an additional indicator to fully quantify the performance level of buildings under seismic loading. As part of the study, a more refined framework for performance-based design and assessment accounting for residual deformation in the definition of performance, by defining a performance matrix which combines both maximum and residual response indices. A direct displacement-based design approach which includes an explicit consideration of the expected residual deformations has also been implemented [Christopoulos and Pampanin, 2004]. Building on the aforementioned proposed framework, extensive numerical analyses have been carried out by Garcia and Miranda[2005] to propose an empirical relationship to evaluate the ratios of residual displacement demand to the peak elastic displacement demand for SDOF systems with known strength ratios. It has been observed that residual displacement ratios exhibit larger levels of record-to-record variability when compared to peak inelastic displacements.

With the recent developments of probabilistic approaches for performance based earthquake engineering, considering the uncertainties on the seismic hazard and on the structural capacities, design objectives can be described in the form of fragility curves representing the probabilities of exceedence of different damage states for various seismic intensity levels.

In this contribution, as part of the development of a refined framework for performance based seismic design and assessment procedures, basic concepts for a probabilistic formulation of the performance based matrix are presented, along with numerical examples on a SDOF system and suggestions for definition of joined maximum-residual performance objectives associated with a targeted probability of exceedence.

2 DEVELOPMENT OF A 3-DIMENSIONAL PERFORMANCE DESIGN OBJECTIVE MATRIX

In response to a recognized urgent need to design, construct and maintain facilities with better damage control, a comprehensive document has been prepared by the SEAOC Vision 2000 Committee [1995], in which, Performance Based Seismic Engineering (PBSE) has been given a comprehensive definition, as *consisting of a set of engineering procedures for design and construction of structures to achieve predictable levels of performance in response to specified levels of earthquake, within definable levels of reliability* and the document includes interim recommendations. Within this proposed framework, expected or desired performance levels are coupled with levels of seismic hazard by performance design objectives as illustrated by the Performance Design Objective Matrix shown in Figure 1.

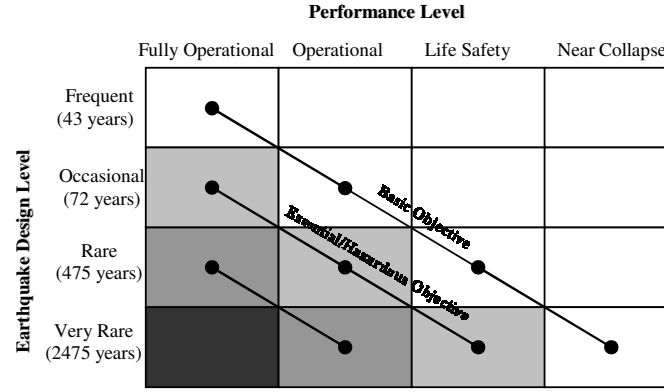


Figure 1. Seismic Performance Design Objective Matrix [SEAO Vision 2000, 1995]

As part of the recently proposed framework for a more comprehensive performance-based seismic design and assessment approach of structures [Pampanin et al., 2002], the concept of a 3-dimensional performance-based matrix has been introduced (Fig. 2), formed by three parameters residual drift, RD, maximum drift, MD, and seismic intensity, IM, on the X, Y and Z axes respectively. For a given seismic intensity, the RD-based performance matrix consists of a mask of pre-defined performance domains (on X-Y plane) or Performance Levels, $PL(i,j)$, defined by the combination of maximum drift, MD, (index i) and residual drift RD, (index j). Higher level of residual drift, for a given value, i , of the maximum response parameter, would result in a poorer combined performance level $PL(i,j)$, corresponding to higher level of damage and repair costs.

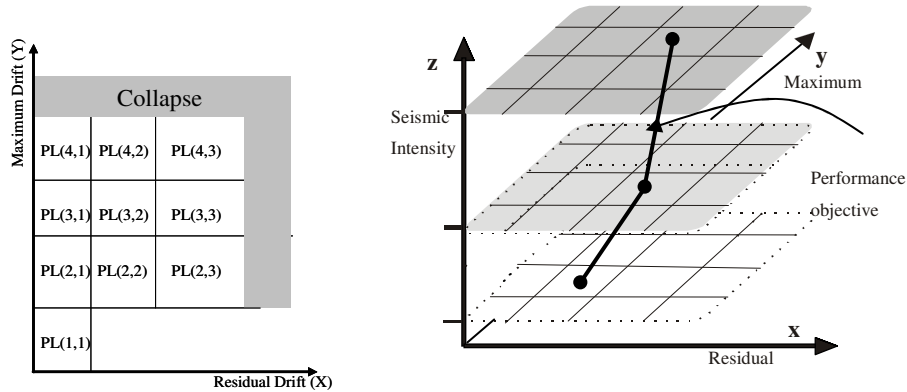


Figure 2: Framework for performance based design and assessment approach including residual deformations: RDDI index and residual-based performance matrix [Pampanin et al. 2002]

Analogous to the Performance Design Objective Matrix developed by the SEAO Vision 2000 document, alternative Performance Objectives associated with different structural systems can be defined within the 3-D performance matrix by connecting a set of performance levels/domains different $PL(i,j)$ belonging to different intensity levels (Fig. 2).

In principle, either a deterministic or probabilistic approach could be used within a performance based design or assessment procedure, with preference to the latter approach when a particular level of confidence of achieving performance objective is of interest. In the next section, a probabilistic formulation of the 3-D performance objective matrix and performance objective is presented.

3 EVOLUTION TOWARDS A PROBABILISTIC FORMULATION OF PERFORMANCE BASED MATRIX CONCEPT

A probabilistic framework for performance-based design and assessment evaluation has been recently proposed by the Pacific Earthquake Engineering Research Center (PEER) [Cornell, 2000]. The whole PBEE process of defining the global performance objectives, in terms of annual probabilities of socio-economic decision variables being exceeded in a seismic hazard environment of the urban region and site under consideration, is decoupled into smaller and easily definable steps. The PEER performance-based design framework utilizes the total probability theory to de-aggregate the problem into several interim probabilistic models (namely seismic hazard, demand, capacity and loss or performance models), to account for the randomness and uncertainty in a more rigorous way. In this study, the development of demand and corresponding capacity models with reference to the performance based matrix concept is discussed.

The Mean Annual Frequency (MAF) of an Engineering Demand Parameter (EDP) exceeding a limit value y can be written as [Cornell 2000]:

$$v_{EDP}(y) = \int G_{EDP|IM}(y|x) d\lambda_{IM}(x) \quad (1)$$

where $\lambda_{IM}(x)$ represents the mean annual frequency of exceedence of seismic hazard intensity measure x in a given seismic hazard environment. The total probability theorem used in the PEER framework equation basically assumes that all the interim models are statistically independent. Hence, the mutual independence of the components is a requirement to guarantee that the demand model is 'sufficient'.

It should be noted that all the interim models are handling only one parameter conditioned on one other parameter. The demand models reported in literature have typically consisted of prediction of the probability of exceeding a given value of one EDP for a given level of Intensity Measure (IM). When implementing the concept of a joined performance-based matrix, the performance levels are defined using a pair of EDPs, i.e. residual and maximum deformations. Hence, a new Probabilistic Seismic Demand Model (PSDM) relating the effects of the selected IM to two EDPs has to be developed.

4 PROBABILISTIC SEISMIC DEMAND MODEL FOR PERFORMANCE MATRIX

The probabilistic assessment of seismic structural performance of a given structure for a given seismic environment is performed using suitable *probabilistic seismic demand model* (PSDM)s which represent the relationship between EDPs and ground motion IMs. Jankovic and Stojadinovic [2004] have critically examined the effects of using alternative IM-EDP pairs for R.C. frame structures and recommended appropriate PSDMs based on their "sufficiency" (i.e. EDP conditioned on IM is independent of magnitude and source-to-site-distance parameters of the earthquake) as well as on their "efficiency" (i.e. smaller dispersion of EDP given IM).

4.1 Joint Probabilistic formulation for combined residual and maximum deformations

Considering the parameters of the PSDM as continuous Random Variables (RV), the uncertainty involved in the prediction of the values of EDPs can be accounted for by associating suitable probability distributions to the RVs. Let us consider three RVs, X , Y and Z corresponding to residual drift (RD), maximum drift (MD) and seismic intensity measure (IM), respectively. Let their individual (marginal) Probability Density Function (PDF) be $f_X(x)$, $f_Y(y)$ and $f_Z(z)$. Hence, at any seismic intensity level, the probability of joint occurrence of the two RVs (X, Y) with values corresponding to a performance level (PL(i,j)) could be expressed by the joint PDF $f(x, y)$ which may be defined as

$$f_{X,Y}(x, y) dx dy = P(x < X \leq x + dx, y < Y \leq y + dy) \quad (2)$$

where the interval between the values of x and $x+dx$ and y and $y+dy$ define the domains on the residual drift axis and on the maximum drift axis respectively corresponding to a PL(i,j). Geometrically the function $f_{X,Y}(x,y)$ is represented in 3 dimensions by a surface above the (x,y) plane with RD and MD along the x and y axes and whose range is the set of probability values corresponding to the ordered pairs of (x,y) in its domain as shown in Fig 3 (a). The probability associated with a performance level, say PL(2,2) is represented by volume *beneath* this surface and is illustrated in Fig.3.

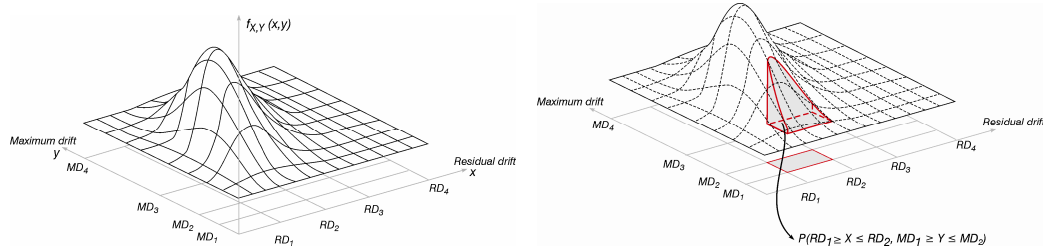


Fig. 3. Joint probability density function over a performance domain

If RD and MD were statically independent, using the multiplication rule the joint PDF could be simply expressed as the product of marginal PDFs of the two RVs and given as

$$f_{X,Y}(x,y) = f_X(x) f_Y(y) \quad (3)$$

Simple statistical measures from regression analysis are helpful to study if the variables show any possible relationship between them. As reported in previous studies in literature [Pampanin et al, 2002] summarized in the previous paragraph, RD and MD have shown a different degree of correlation at various intensity levels, thus impairing the hypothesis of statistically independent variables. Hence, it has been decided and proposed in this study to use a single bivariate joint PDF to describe the joint occurrence of a pair of RD and MD over a performance domain.

Following the observed trends of EDPs, typically used in PSDMs and following a lognormal distribution, a bivariate lognormal PDF has been adopted when referring to a performance domain at a given intensity level. A bivariate log-normal distribution for the joint distribution of residual drift (X) and maximum drift (Y) with the joint PDF may be written as

$$f_{X,Y}(x,y) = \frac{0.5}{xy\pi\zeta_X\zeta_Y\sqrt{1-\rho^2}} * \exp\left\{-\frac{0.5}{1-\rho^2}\left[\frac{(\log x - \lambda_X)^2}{\zeta_X^2} - \frac{2\rho(\log x - \lambda_X)(\log y - \lambda_Y)}{\zeta_X\zeta_Y} + \frac{(\log y - \lambda_Y)^2}{\zeta_Y^2}\right]\right\} \quad (4)$$

Where $\lambda_X, \lambda_Y, \zeta_X$ and ζ_Y are the location and scale parameters of the marginal PDF of X (residual drift) and Y (maximum drift), respectively. The parameter ρ forms a linear correlation coefficient between the two variables.

5 DEVELOPMENT OF PERFORMANCE OBJECTIVES FROM 3-D PERFORMANCE MATRIX: A PROBABILISTIC APPROACH

The probabilistic approach to define target performance objectives according to the 3-dimensional performance matrix concept is presented and discussed. The procedure involves seismic response analysis from which the probabilities of exceedence of performance levels are evaluated.

5.1 Probabilistic procedure adopted on 3-dimensional performance-based matrix

A PSDM is appropriately selected and seismic response analyses are carried out for the chosen structural system for a suite of earthquake records varying the levels of IM. The EDPs the residual and

maximum drifts at every level of IM are analysed for their statistical parameters to describe the joint PDF. These data pairs correspond to a single 2-D performance domain.

The probability of achieving a PL(i,j) specific to certain domain of RD and MD is obtained by performing double integration over the joint PDF as in Eq. 3, with respective values of the variables as upper and lower limits of integration. For example, the probability associated with PL (2,2) is given by

$$\int_{MD_1}^{MD_2} \int_{RD_1}^{RD_2} f_{X,Y}(x,y) dx dy \quad (6)$$

The probability of exceeding PL(i,j), e.g. PL(2,2), is given by the volume under shaded portion of the surface area as shown in Figure 4, which may also be expressed as 1 minus the probability of reaching or being within PL(2,2)). At this stage, it may be of interest to know the contribution to the probability distribution by alternative pairs of RD and MD. As shown in Fig. 4, the zone A may be interpreted as contribution mostly governed by MD, zone B as that mostly governed by RD and zone C as that governed by both parameters.

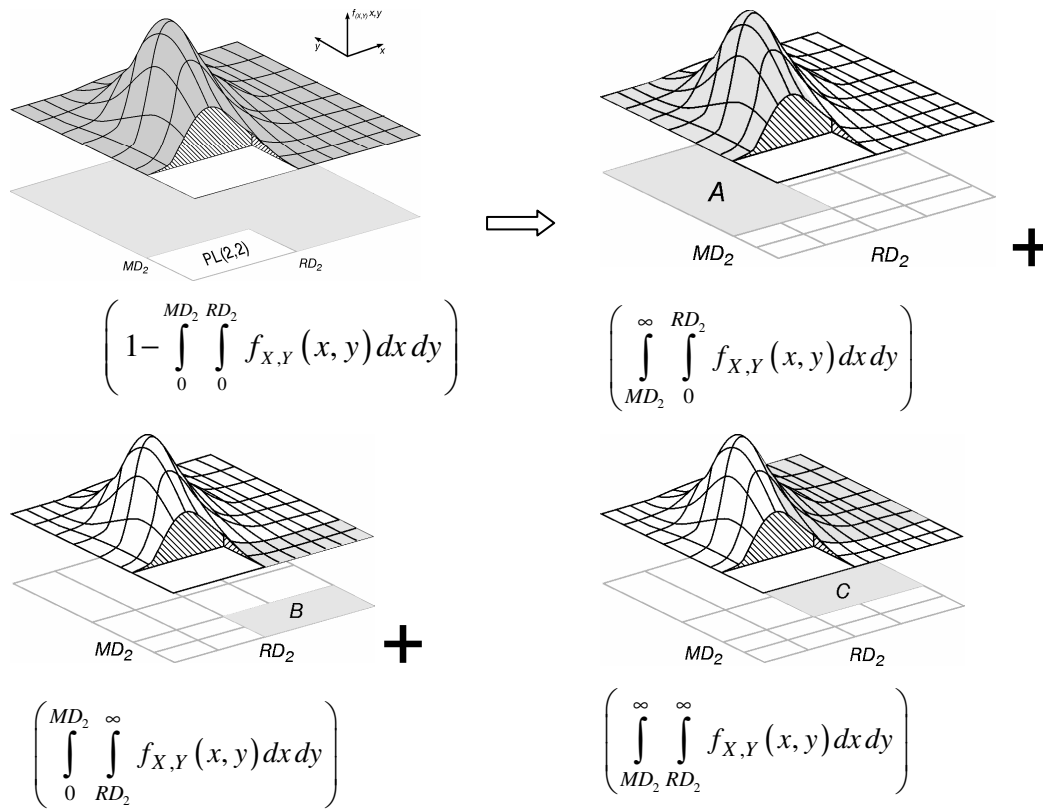


Fig. 4. Probability of exceedence of PL(2,2) and the contributions from the response parameters

The total probability of exceedence is evaluated for (PL(i,j)) on a performance domain associated with a given level of IM and is repeated for the considered intensity levels. Fragility curves for PL (i,j) is fitted to the above computed data. Finally, the desired performance objectives with the associated probability are established by connecting various performance levels with increasing seismic hazard levels.

6 NUMERICAL EXAMPLES ON PERFORMANCE EVALUATION OF SDOF SYSTEMS

6.1 Analytical modelling of single degree of freedom systems

An equivalent Single Degree of Freedom (SDOF) system, representing an 8 storey reinforced concrete frame building designed according to a displacement based design approach with a target maximum interstorey drift of 2.5% and a peak spectrum acceleration of 0.5 g [Priestley, 1998] have been considered. The dynamic properties of the SDOF systems can be obtained from Pampanin et al. [2002]. The nonlinear responses were assumed to follow two hysteretic behaviours such as Elasto-Plastic (EP) model with zero post yield stiffness and Takeda (TK) model with 0.02 as post yield stiffness ratio. The hysteresis coefficients describing the unloading and reloading stiffness behaviour, (typically known as α and β) were taken as 0.3 and 0.2, respectively. Two additional SDOF systems with alternative strength ratios values (0.8 and 1.2 times the reference system) was also considered for the analyses.

6.2 Seismic Response Analysis

A series of inelastic time-history analyses using a selected suite of earthquake records were performed on the SDOF systems modelled in Ruaumoko [Carr, A.J. 2003] based on a lumped plasticity approach. A total of thirty earthquake ground motions were utilised in this study. They were extracted from two sources: the database used by Pampanin et al. [2002] and the Pacific Earthquake Engineering Research data base [PEER, 2000]. The records represent magnitude ranging from 6.5 to 7.2, closest distance to fault rupture varying from 15 km to 30 km and soil category C and D (according to NEHRP provisions[1997]). The Spectral velocity (S_v) corresponding to the initial period of the SDOF system is chosen as IM for the building and is scaled to ten different intensity levels from 0.05 m/s to 0.5 m/s. Three systems were analysed for 30 records with 10 different intensity levels. This would result in 90 pairs of residual and maximum drift ratios within a 2 dimensional performance domains in the X-Y plane. The 3-D performance matrix can be visualised with 10 mono-intensity 2-D performance domains.

A multivariate linear regression analysis was performed to check the mutual independence of the ground motion parameters (such as intensity, magnitude and distance) on EDPs [Mackie and Stojadonovic, 2003]. It was observed that the regression coefficients corresponding to the magnitude and distance variables were not as significant as that corresponding to the IM ensuring the sufficiency of the model to relate independently S_v with EDPs.

6.3 Limit states for engineering demand parameters for various performance levels

Definition of limit states plays a significant role in the development of performance levels with associated probability in the form of fragility curves. Extensive research investigations and publications available in literature have proposed different limit states mostly referring to the maximum transient responses. Large permanent (residual) deformations observed in bridge piers after the 1995 Hyogo-Ken-Nambu earthquake triggered off major research on residual deformations. Based on early research works, few publications have pointed out the need to check permanent (residual) deformations in the structural system [NEHRP, 1997, Kawashima, 1997] and provided indicative values for checks. In this study, referring to the previous research work [Pampanin et al., 2002] and draft guidelines of AIJ [2004], tentatively values for the limit states based on residual drift, RD, are taken as 0.2%, 0.4%, 0.6% and 1.0% while, more traditional values for the limit states based on maximum drift, MD, are considered as 0.5%, 1.0%, 2.0% and 4.0%. The damage states can be referred to as “serviceability”, “repairable damage”, “irreparable damage” and “collapse prevention”.

6.4 Development of fragility curves

A statistical distribution is fitted to the data pairs with residual and maximum drift on every performance domain obtained from each intensity level. For example, a plot of residual and maximum drifts for Takeda system at intensity level ($S_v = 0.15$ m/s) has been shown in Fig.5. The lognormal parameters (mean and standard deviation) and the correlation coefficient for the set of RD and MD

data are calculated to construct bivariate lognormal joint PDF. The correlation between the variables was more evident at high intensity levels than at lower intensity levels.

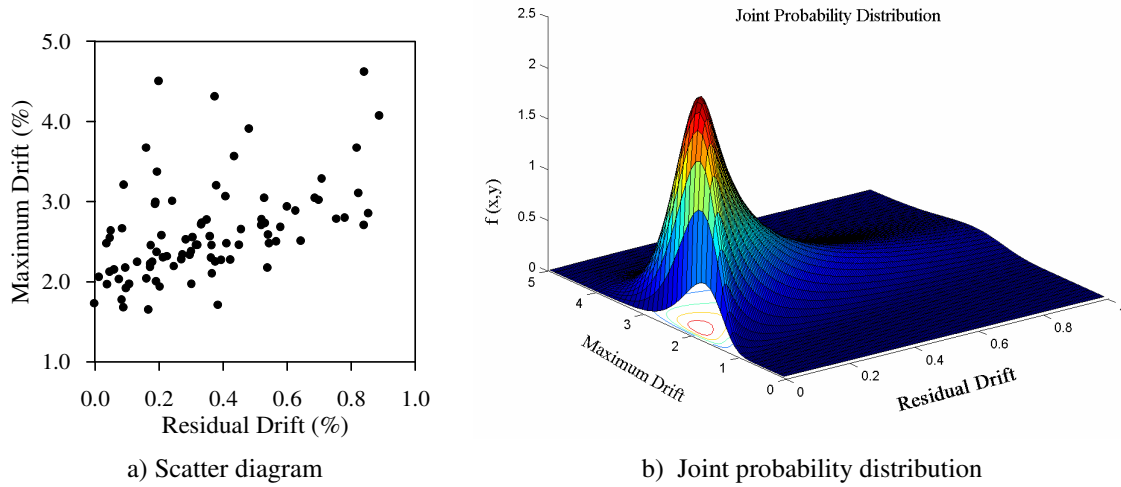


Fig. 5. Distribution of residual and maximum drift

The total probability of exceedence of a PL is obtained using Eq. 7 and by adopting the procedure described in section 3.1. A smooth fragility can be fitted to the data points assuming a lognormal distribution. Fig. 6 shows the probabilities of exceedence corresponding to PL(4,2) for EP and TK systems. In EP systems higher probability of exceedence of the PL is governed by RD and lower probability of exceedence is governed by MD when compared to TK systems. Fig 6 (a) and (b) illustrates the contributions referred to A, B and C to the total probability of exceedence which shows substantial variation between the two systems. The contributions of residual (B) and combined (C) zones is predominant in EP system where as the contribution from maximum drift (A) zone is more in the case of TK system. From this, the influence of the response parameters to the probability of exceedence of a PL can be recognised.

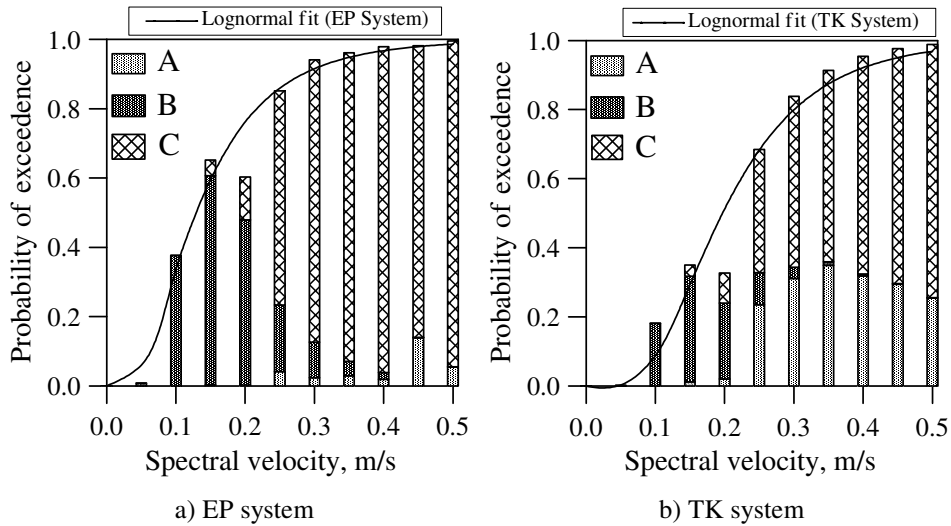


Fig. 6. Contributions to the total probability of exceedence for PL(4,2), zones A,B and C

As shown in Fig. 2, in a design or assessment phase, the performance objectives can be obtained by connecting the performance levels with targeted probability of achieving them. Fig. 7a shows the visualization of typical performance objectives when adopting fragility curves. Major increase in the confidence of the design could be for example achieved by targeting a defined level of probability of exceedence of different performance levels for increase levels of intensity. Within a complete probabilistic formulation (including the seismic hazard model), a “uniform risk” design approach could be suggested to consist in targeting same probability of exceeding PLs belonging to a predefined

performance objective, as shown in solid line (UR) in Fig. 7a. Alternatively, a variable level of acceptable probability of exceedence (i.e. multiple risk in a general formulation) associated to higher intensity and performance levels can be adopted in the design phase (as indicated with a dashed line (MR)). For example, the line (MR) connects PL(2,2), PL(3,3) and PL(4,4) with respective probabilities of achieving them of 40%, 30% and 20%.

The significance of including the residual deformations as a complementary damage indicator parameter in addition to maximum deformation indexes when assessing the actual performance level is illustrated in Fig. 7b, where fragility curves obtained for PLs corresponding to maximum drift limits ($i=3$ and 4) combined with residual drift limits (for $j=1,2,3$) are presented. It can be seen that the fragility curves for PL(3,2) and PL(3,3) represent higher damage state than PL(4,1). In other words, for a chosen intensity level, e.g. $S_v=0.1$ m/s, the probability of exceedence of PL (3,2) and PL(3,3) is lower than that corresponding to PL(4,1). Thus, although being subjected to similar maximum drift demands, the systems should be assigned substantially different levels of performance, depending on the value of the residual response indices.

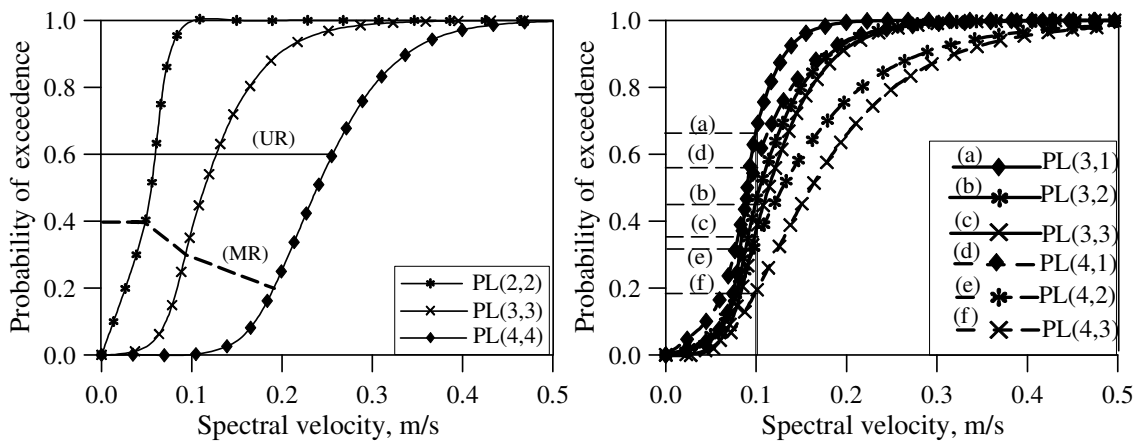


Fig. 7. a) Fragility curves and performance objective; b) influence of residual drift on fragility curves

7 CONCLUSIONS

A probabilistic formulation of a 3-dimensional performance matrix combining maximum and residual deformation to define performance levels and performance objectives at increasing level of seismic intensity has been presented. Bivariate lognormal probability density functions are used to describe the joint occurrence of residual and maximum deformations within defined performance domains or levels PL(i,j). Fragility curves representing the probabilities of achieving or exceeding different maximum-residual performance levels are derived. Numerical examples on the seismic response of SDOF systems have been provided. It has been observed that the contributions of the maximum or residual response parameters to the total probability of exceedence of a performance level, PL(i,j) were largely influenced by the hysteresis models. Given the intensity level, EP systems predict higher probability of exceedence of a PL governed by RD and lower probability of exceedence governed by MD when compared to TK systems. The amount contributed by each response parameters would be an important factor to suggest mitigation strategy in a design phase as well as suitable retrofitting interventions to achieve higher performance. In conclusion, preliminary suggestions for a “uniform risk design” (or controlled “multiple risk”) approach, whereby targeted probability of exceeding predefined Performance Levels, at increasing intensity levels, are controlled in the design phase, has been given.

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